Modal Logics for Multigranulation Rough Set Models

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1 Abstract

Rough set theory (RST), initiated by Pawlak, provides a foundational framework for modelling vagueness and uncertainty through approximation operators induced by indiscernibility relations. From its inception, the modal character of rough set approximations has been evident, with Pawlak's classical model aligning naturally with the modal system S5. As rough set theory has evolved through a range of generalisations - such as approximation spaces based on non-equivalence relations, multi-granulation models, information systems, and multiple-source approximation systems - the limitations of standard modal semantics have become increasingly apparent, motivating the development of richer and more flexible logical frameworks. This tutorial presents a systematic introduction to modal logics whose semantics are grounded in rough set theory, encompassing both classical foundations and recent developments. We begin by revisiting the modal interpretation of lower and upper approximation operators in Pawlak's rough set model, and then move beyond the S5 setting to accommodate generalized approximation spaces.

Rough set frameworks involving multiple relations appear at the very outset of the subject, as evidenced by early contributions of Pawlak, Orłowska, Farinas del Cerro, Rauszer, and others. Subsequent proposals on multi-granulation rough set models and the associated notions of strong (pessimistic) and weak (optimistic) approximation operators have opened new directions in this line of research. We demonstrate how these operators can be captured within the basic modal language through distinct semantic clauses, leading to sound and complete axiomatizations over broad classes of models, without requiring any extension of the underlying language.

Another part of the tutorial covers the role of relative accessibility relations, which arise naturally when approximations are considered relative to attribute sets or information sources. We discuss modal languages with parameterised

modalities, information structures, and multiple-source information structures, and explain how algebraic operations on parameter sets interact with semantic constraints on accessibility relations.

The tutorial also addresses temporal information systems, in which information systems evolve over time and approximation operators become time-dependent. Reasoning about such dynamic settings requires modal frameworks capable of capturing changing granularity, evolving attribute-values, and persistence of information across time. We outline how temporal and multi-modal extensions of rough set semantics can be handled within a unified logical perspective.

The tutorial is designed to be accessible to researchers in rough set theory while also offering new insights to logicians interested in modal semantics beyond standard Kripke frames.

2 Main Objectives

The tutorial has the following objectives.

2.1 Conceptual grounding and Generalised approximation semantics

To clarify the intrinsic connection between rough set approximation operators and modal logic, beginning with Pawlak's approximation spaces and their correspondence with the modal system S5.
To present modal semantics corresponding to generalised approximation spaces, including those based on relations that are not equivalence relations.

2.2 Multi-granulation and multiple-source reasoning

To present the modal-logical treatment of multi-granulation rough set models, with a focus on strong and weak approximation operators and their semantic and axiomatic characterisations.

2.3 Relative and parameterised modalities

To explain modal logics with relative accessibility relations arising from attribute-based and source-based approximations, including information structures and multiple-source information structures.

2.4 Temporal information systems

To present time-indexed information systems, the evolution of attributes and approximations, and modal reasoning about changing information and granularity.

3 Relevance to the Rough Sets Community

The proposed tutorial is directly aligned with core themes of rough set theory and its ongoing development. While rough sets have traditionally relied on algebraic and set-theoretic tools, modal logic provides a complementary and unifying perspective that allows approximation mechanisms to be analysed within a precise semantic and proof-theoretic framework. In particular, modal logic offers formal languages for reasoning about approximation operators, granulation, and boundary regions; clarifies the structural properties that distinguish different rough set models via axioms and completeness results.

By focusing on both foundational ideas and recent advances, the tutorial aims to equip participants with conceptual and technical tools that can inform future research in rough set theory, knowledge representation, and uncertainty reasoning.

Selected Bibliography

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