GrC Techniques on Simple Undirected Graphs

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Extended Abstract

In this tutorial we present a new formal framework for Rough Set Theory (RST) introduced for the first time in [3, 4], along with various applications to graph theory recently achieved in [2, 5]. By emphasizing intuition, our purpose consists of explaining the mathematical foundation of several notions and techniques of RST based on Pawlak's information tables [8] via examples and potential applications rather than technical formalism.

The tutorial is organized into two main parts. The first part introduces the conceptual background needed to understand the proposed approach. Given a Pawlak's information table $\mathcal{I} = (U, \mathrm{Att}, \mathrm{Val}, F)$, we show how to associate with it different kinds of set systems, set operators and binary set relations on the attribute set and, at the same time, we interpret various related results obtained in [3, 4, 6].

More specifically, via the classical Pawlak's indiscernibility relation we induce an equivalence relation on the powerset of Att, that we call *indistinguishability relation*. This allows us to group together subsets that generate the same informational content and to study their behavior from both a local and a global perspective.

We discuss in detail its mathematical properties and their interpretations in applied contexts. In particular, indistinguishability classes are union-closed and therefore admit a maximum element, called *maximum partitioner*, representing the largest attribute subset inducing a given indiscernibility relation. The family $\mathrm{MAXP}(\mathcal{I})$ of all maximum partitioners, when ordered by inclusion, forms a complete lattice, which provides a global representation of the indiscernibility relations induced by the information table (the so-called *granular partition lattice* [3]). We also propose an alternative but essentially equivalent relational counterpart to the granular partition lattice through the formalization of the notion of *functional dependence* from database theory [1].

In addition, we focus our attention also to the collection $\mathrm{MINP}(\mathcal{I})$ of the *minimal partitioners* of $\mathcal{I}$, i.e. the minimal attribute subsets of each indistinguishability class, by stating their main properties and their non-trivial relationship with maximum partitioners.

As can be seen from the above description, the proposed framework naturally supports two complementary levels of analysis. On the one hand, it allows a *micro-granular* investigation, focused on local information carried by attribute subsets; on the other hand, it provides a *macro-granular* perspective, capturing the global organization of indiscernibility relations through lattice structures. The interaction between these two levels plays a central role in understanding how local information aggregates into global patterns within a Pawlak's information table [4].

The second part of the tutorial is devoted to the application of the aforementioned mathematical setting to graph theory, with the specific purpose of determining new structural properties of simple undirected graphs. While early connections between graph theory and GrC were proposed by Stell [9] in the context of qualitative spatial reasoning, our approach (see [2, 5]) exploits RST-based tools as a systematic methodology for the analysis of structural and combinatorial properties of graphs. Indeed, every simple graph can be represented by different matrices, such as adjacence, distance, incidence, or Laplacian matrices. Each of these representations induces a distinct Pawlak's information table, leading to non-equivalent granular descriptions of the same graph. We discuss analogies and differences among these viewpoints, showing how each choice emphasizes specific graph-theoretic features.

Accordingly, GrC tools admit a direct translation into meaningful structural properties of graphs, depending on the selected matrix representation. For instance, when considering the Pawlak's information table induced by the adjacence matrix, the indiscernibility relation captures a local symmetry already studied by P. Erdös, A. Rényi [7]. When the distance matrix is considered instead, reducts coincide with the *resolving sets* of the graph and their minimum cardinality yields its *metric dimension* [5].

We conclude by presenting several results that explicitly connect graph-theoretic properties with the combinatorial behavior of maximum and minimal partitioners and related theoretical open problems, thereby demonstrating the applicability and flexibility of the proposed granular framework in graph theory and, more broadly, in its areas of application.

Main Goals of the Tutorial

The main goal of this tutorial is to establish a solid connection between a new mathematical formalization of RST and graph theory, showing how GrC tools can be systematically used to describe and analyze structural properties of simple undirected graphs. Overall, focusing on intuition, examples and applications rather than technical proofs, the tutorial illustrates how RST-based granular methods provide a flexible and systematic approach to the study of graphs, offering tools that are applicable not only within graph theory itself, but also in interdisciplinary contexts where graphs are used to model complex systems, such as computer science, engineering, robotics and chemistry.

Finally, the tutorial also aims to present some technical challenges to be addressed in successive research, thereby demonstrating the relevance and the richness of this new research field.

Relevance of the Tutorial of the RST Community

The tutorial is relevant for the RST community as it contributes, on the one hand, to the ongoing effort of providing a rigorous and unifying mathematical foundation for GrC and RST techniques and, on the other hand, to create a bridge between RST and combinatorics. The ideas we intend to convey through our tutorial make it possible to interpret several existing constructions as instances of common underlying patterns, thereby clarifying their mutual interplay and conceptual scope. From an interpretative viewpoint, the proposed framework supports the transfer of insights obtained from specific case studies to more general settings, facilitating the identification of meaningful analogies across different RST-based models. Moreover, by explicitly connecting local and global levels of analysis, the tutorial highlights structural aspects that are often treated separately in the literature. Finally, the interaction with graph theory opens new perspectives for applying RST tools and suggests novel research directions within and beyond the traditional boundaries of the field.

Keywords: Pawlak's Information Tables · Granular Partition Lattice · Graphs

References

1. W. W. Armstrong, Dependency structures of data base relationships, *Information Processing* 74 (North-Holland, Amsterdam, 1974) 580–583.
2. C. Bisi, F.G. Infusino, Representation Theorems for Simplicial Complexes and Matroidal-Like Properties of Minimal Partitioners, Advances in Applied Mathematics, 162, 2025, 102778.
3. G. Chiaselotti, D. Ciucci, T. Gentile, F. Infusino, The Granular Partition Lattice of an Information Table. *Information Sciences* 373 (2016), 57–78.
4. C. Bisi, G. Chiaselotti, D. Ciucci, T. Gentile, F. Infusino, Micro and Macro Models of Granular Computing induced by the Indiscernibility Relation. *Information Sciences*, 388–389 (2017), 247–273.
5. G. Chiaselotti, T. Gentile, F. Infusino, P. A. Oliverio, The Adjacency Matrix of a Graph as a Data Table. A Geometric Perspective. *Annali di Matematica Pura e Applicata*, Vol. 196, No. 3, (2017), 1073–1112.
6. G. Chiaselotti, T. Gentile, F. Infusino, Granular Computing on Information Tables: Families of Subsets and Operators, *Information Sciences* 442–443 (2018), 72–102.
7. P. Erdös, A. Rényi; Asymmetric graphs, *Acta Mathematica Hungarica*, Vol.14, Issue 3–4, pp. 295–315, 1963.
8. Z. Pawlak, Rough sets. Theoretical Aspects of Reasoning about Data. Kluwer Academic Publisher, 1991.
9. J. G. Stell, Granulation for Graphs, Sp. Inf. Th., *Lecture Notes in Computer Science*, Volume 1661, 1999, 417–432.